

## F20 MAT 101S Assignment E

### Factoring Polynomial Functions (Content from Unit 4 in MAT 101)

| Zeros                                                                                                                                                                      | Factors                                                                                                                                                                                                                                                                           | How do zeros & factors relate?                                                                                                                                                |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> <li>• <math>x</math>-values that make the function equal 0</li> <li>• <math>x</math>-values of the <math>x</math>-intercepts</li> </ul> | <ul style="list-style-type: none"> <li>• two expressions that multiply together</li> <li>• any polynomial that divides evenly into <math>f(x)</math></li> <li>• of the form <math>(x - \text{"zero of polynomial"})</math></li> <li>• of the form <math>(x - c)</math></li> </ul> | <ul style="list-style-type: none"> <li>• If 2 is a zero, then <math>(x - 2)</math> is a factor.</li> <li>• If -1 is a zero, then <math>(x + 1)</math> is a factor.</li> </ul> |

**Example - Factor:**  $f(x) = 2x^3 - 5x^2 - x + 6$       **\*Notice this is in function notation.**

**Step 1: Find all possible zeros of the function.** To narrow our search for zeros, we can use information from the function to figure out numbers that could possibly be a zero.

**P** is the **constant** in the function. In our case, P is 6.

**Q** is the **leading coefficient**. In our case, Q is 2.

List all factors of P: 1, 2, 3, 6. (What multiplied by what makes 6? Use all of those numbers.)

List all factors of Q: 1, 2. (What multiplied by what makes 2? Use all of those numbers.)

Use  $\frac{P}{Q}$  to find all possible zeros. Remember the zero could possibly be negative or positive.

$$\frac{P}{Q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2} = \pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{3}{1}, \pm \frac{6}{1}, \pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{3}{2}, \pm \frac{6}{2}$$

$$\text{Simplify} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 3 \quad \text{Omit Duplicates} = \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 2, \pm 3, \pm 6$$

So, this list gives us a list of all POSSIBLE zeros of the function above.

How do we find the ACTUAL zeros?

**Step 2: Use synthetic division to find a number that gives a ZERO remainder.**

Let's start with one of the possible zeros in the list. It doesn't matter which one you pick to try first.

Use the process for synthetic division previously taught.

**Let's try 1 as a possible zero.**

$$\begin{array}{r|rrrr} 1 & & 2 & -5 & -1 & 6 \\ & & & 2 & -3 & -4 \\ \hline & & 2 & -3 & -4 & 2 \end{array}$$

\* So, our remainder was 2, which is not zero.

Therefore, 1 is not a zero of  $f(x)$ .

**Let's try -1 as a possible zero.**

$$\begin{array}{r|rrrr} -1 & & 2 & -5 & -1 & 6 \\ & & & -2 & 7 & -6 \\ \hline & & 2 & -7 & 6 & 0 \end{array}$$

**Bingo! Our remainder was zero.**

Therefore,  $-1$  is a zero of  $f(x)$ , and  $(x + 1)$  is a factor of  $f(x)$ .

**Step 3: Rewrite the equation with the new factor.**

Let's rewrite  $f(x)$  with our new factor. The format will be  $(x - \text{"zero"})(\text{quotient})$ .

$$f(x) = (x + 1)(2x^2 - 7x + 6)$$

**Step 4: Using a method of factoring to factor the trinomial factor.** Now that we have a semi-factored form, let's see what else we can do:

$$f(x) = (x + 1)(2x^2 - 7x + 6)$$

$$2x^2 - 7x + 6 \text{ can be factored even further into } (2x - 3)(x - 2).$$

\*Use grouping to factor.

So, the final factored form is:  $f(x) = (x + 1)(2x - 3)(x - 2)$ .

If we set each of these factors = 0, we get the **zeros or solutions to  $f(x) = 0$** :  $-1, \frac{3}{2}, 2$ .

1a. [2 pts] Show that 2 is a zero of  $f(x) = 4x^3 - 12x^2 + 5x + 6$  using synthetic division.

$$\begin{array}{r|rrrr}
 2 & 4 & -12 & 5 & 6 \\
 & & 8 & -8 & -6 \\
 \hline
 & 4 & -4 & -3 & 0
 \end{array}$$

1b. [2 pts] Write  $f(x)$  in the form  $(x - c)(Q(x))$ :  $(x - 2)(4x^2 - 4x - 3)$

1c. [3 pts] Factor  $Q(x)$  using any method you choose.

$$\begin{aligned}
 &(4x^2 - 4x - 3) \\
 &4x^2 - 6x + 2x - 3 \\
 &2x(2x + 1) - 3(2x + 1) \\
 &(2x - 3)(2x + 1)
 \end{aligned}$$

Factored form of  $Q(x)$ :  $(2x - 3)(2x + 1)$

1d. [3 pts] Write the factored form of  $f(x)$ .  $f(x) = (x - 2)(2x - 3)(2x + 1)$

**Definition: A factor of an integer is any number that divides that integer evenly (zero remainder).**

2a. [2pts] List the factors of 15: 1, 3, 5

2b. [2pts] List the factors of 20: 1, 2, 4, 5

2c. [2pts] List the factors of 24: 1, 2, 3, 4, 6, 8, 12

2d. [4 pts] If you wanted to create all the possible fractions you could given 1, 2, 5, and 10 as a list of numbers you must use for the numerator and 1, 2, and 4 as a list of numbers you must use for the denominator, what would be your list of fractions that would be created using these numbers.

I'll get you started:  $\frac{1}{1}, \frac{2}{1}, \frac{5}{1}, \frac{10}{1}, \frac{1}{2}, \frac{2}{2}, \frac{5}{2}, \frac{10}{2}, \frac{1}{4}, \frac{2}{4}, \frac{5}{4}, \frac{10}{4}$

Then, simply each fraction. This would give:

$1, 2, 5, 10, \frac{1}{2}, \frac{5}{2}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2}, \frac{5}{2}$

3. Follow the step-by-step guide below to help you write the following function in factored form. Then, solve  $f(x) = 0$ .

$$f(x) = 2x^3 - 3x^2 - 3x + 2$$

[2 pts] Part A: Find all the possible zeros.

What is P? 2

What is Q? 2

What is  $\frac{P}{Q}$ ? Be sure to simplify.

Write the list of all possible zeros:  $-2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2$

[2 pts] Part B: Find one actual zero.

Use synthetic division to test the possible zeros. How do you know if it is an actual zero?

|    |   |    |    |    |
|----|---|----|----|----|
| -1 | 2 | -3 | -3 | 2  |
|    |   | -2 | 5  | -2 |
|    | 2 | -5 | 2  | 0  |

[2 pts] Part C: Rewrite the function by turning the actual zero into a factor and the coefficients into a trinomial factor.

Rewrite the function with the new factor:  $f(x) = \underline{(x + 1)(2x^2 - 5x + 2)}$

[2 pts] Part D: Factor the trinomial factor (the quotient from above).

Factor the trinomial factor in the function above:

$$(2x^2 - 5x + 2)$$

$$(2x^2 - 4x - x + 2)$$

$$(2x(x - 2) - 1(x - 2))$$

$$(2x - 1)(x - 2)$$

[3 pts] Part E: Write the final factored form of the function. Remember:  $f(x) = (x - \text{zero})(\text{factored quotient})$ .

Final factored form:  $f(x) = \underline{(x + 1)(2x - 1)(x - 2)}$

[3 pts] Part F: Set each of the binomial factors equal to zero and solve each for  $x$  to find the solutions to  $f(x) = 0$ .

What are the solutions to  $f(x) = 0$ ?  $x = -1, \frac{1}{2}, 2$

4. [15 pts] Write the following function in factored form. Then, solve  $f(x) = 0$ .

Write your final answers at the bottom.

$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

P = 20, Factors = 1, 2, 4, 5

Q = 2, Factors = 1, 2

Possible zeros:  $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 5, \pm \frac{5}{2}, \pm 10, \pm 20$

|    |   |     |     |     |
|----|---|-----|-----|-----|
| -4 | 2 | -3  | -39 | 20  |
|    |   | -8  | 44  | -20 |
|    | 2 | -11 | 5   | 0   |

$$f(x) = (x - 4)(2x^2 - 11x + 5)$$

$$2x^2 - 11x + 5$$

$$2x^2 - 10x - x + 5x$$

$$2x(x - 5) - (x - 5)$$

$$(2x - 5)(x - 5)$$

$$f(x) = (x + 4)(2x - 5)(x - 5)$$

Answers:

Write the list of all possible zeros:  $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 5, \pm \frac{5}{2}, \pm 10, \pm 20$

Final factored form:  $f(x) = (x + 4)(2x - 5)(x - 5)$

What are the solutions to  $f(x) = 0$ ?  $x = -4, \frac{5}{2}, 5$

5. [15 pts] Write the following function in factored form. Then, solve  $f(x) = 0$ .

Write your final answers at the bottom.

$$f(x) = 2x^3 - x^2 - 22x - 24$$

P = 24, Factors = 1, 2, 3, 4, 6, 8, 12

Q = 2, Factors = 1, 2

Possible zeros:  $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

|    |   |    |     |     |
|----|---|----|-----|-----|
| -2 | 2 | -1 | -22 | -24 |
|    |   | -4 | 10  | 24  |
|    | 2 | -5 | -12 | 0   |

$$f(x) = (x + 2)(2x^2 - 5x - 12)$$

$$2x^2 - 5x - 12$$

$$2x^2 - 8x + 3x - 12$$

$$2x(x - 4) + 3(x - 4)$$

$$(2x + 3)(x - 4)$$

$$f(x) = (x + 2)(2x + 3)(x - 4)$$

$$f(x) = 0$$

$$x = -2, -\frac{3}{2}, 4$$

Answers:

Write the list of all possible zeros:  $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

Final factored form:  $f(x) = (x + 2)(2x + 3)(x - 4)$

What are the solutions to  $f(x) = 0$ ?  $x = -2, -\frac{3}{2}, 4$

**6. Find the LCM of each set of terms.**

a. [2 pts] 90 and 20

LCM: 180

b. [3 pts]  $4x^2$ ,  $2x$ , & 11

LCM:  $44x^2$

c. [3 pts]  $x + 5$  and  $x - 5$ .

LCM:  $x^2 - 25$

**Definition:** The least common denominator (LCD) of two or more fractions is the least common multiple (LCM) of the denominators.

**Example:** Find the LCD of  $\frac{2}{7}$  and  $\frac{3}{5}$ .

Solution: We are looking for the LCM of the two denominators: 7 & 5. So, the answer is **35**.

**7. Find the LCD of each set of rational expressions.**

a. [2 pts]  $\frac{1}{2}$  and  $\frac{7}{11}$

**22**

b. [3 pts]  $\frac{x}{x+2}$  and  $\frac{7}{x-2}$

$x^2 - 4$

c. [3 pts]  $\frac{1}{x}$ ,  $\frac{2}{x+4}$ , and  $\frac{1}{x^2+4x}$

$x^2 + 4x$

Solving a **RATIONAL EQUATION**: Use this as a reference for the next two problems.

$$\frac{2}{x+4} + \frac{1}{x-4} = \frac{17}{x^2-16}$$

Step 1: Factor all denominators.  $\frac{2}{x+4} + \frac{1}{x-4} = \frac{17}{(x+4)(x-4)}$

Step 2: Find the LCD. The LCD is  $(x+4)(x-4)$ .

Step 3: Multiply each term by a factor equaling 1 to get a common denominator.

$$\frac{2}{x+4} \cdot \frac{(x-4)}{(x-4)} + \frac{1}{x-4} \cdot \frac{(x+4)}{(x+4)} = \frac{17}{(x+4)(x-4)}$$

Step 4: Multiply each term by the LCD:

$$(x+4)(x-4) \left( \frac{2}{x+4} \cdot \frac{(x-4)}{(x-4)} \right) + (x+4)(x-4) \left( \frac{1}{x-4} \cdot \frac{(x+4)}{(x+4)} \right) = (x+4)(x-4) \left( \frac{17}{(x+4)(x-4)} \right)$$

Step 5: Cancel out common factor, eliminating the fractions.

$$\cancel{(x+4)}\cancel{(x-4)} \left( \frac{2}{\cancel{x+4}} \cdot \frac{\cancel{(x-4)}}{\cancel{(x-4)}} \right) + \cancel{(x+4)}\cancel{(x-4)} \left( \frac{1}{\cancel{x-4}} \cdot \frac{\cancel{(x+4)}}{\cancel{(x+4)}} \right) = \cancel{(x+4)}\cancel{(x-4)} \left( \frac{17}{\cancel{(x+4)}\cancel{(x-4)}} \right)$$

Step 6: Write the remaining terms.  $2(x-4) + 1(x+4) = 17$

Step 7: Solve the equation remaining. (This one is not quadratic, but if it is, you would have to factor or use other methods to solve.)

$$2x - 8 + x + 4 = 17 \quad \text{Linear Equation: Solve for } x.$$

$$3x - 4 = 17$$

$$\begin{array}{r} +4 \quad +4 \\ \hline 3x = 21 \\ \hline 3 \quad 3 \end{array}$$

$$x = 7$$

Step 8: Check your answer! Will it make the denominator equal zero?

The LCD is  $(x+4)(x-4)$ .

Plug the possible solution(s) into the LCD & see if it makes one of the factors = 0.

Let's test  $x = 7$  in the LCD:  $(7+4)(7-4) = (11)(3) = 33$  which is NOT zero, so 7 works and is a solution!

Therefore, we can use 7 as our answer.

Final Solution:  $x = 7$

\*Note, some equations are quadratics after eliminating the fractions. Factor and solve. Check EACH possible answer in the LCD to see if it makes it = 0. If one does and one does not, eliminate the one that makes the LCD = 0. You would only have one solution. If BOTH make the LCD = 0, your answer is NO SOLUTION (DNE).

**8. [10 pts] Solve the following rational equation using the previous reference page as a guide.**

$$\frac{2}{x+3} + \frac{5}{x-3} = \frac{37}{x^2-9}$$

$$(x^2 - 9) \frac{2}{x+3} + (x^2 - 9) \frac{5}{x-3} = (x^2 - 9) \frac{37}{x^2 - 9}$$

$$2(x-3) + 5(x+3) = 37$$

$$7x + 9 = 37$$

$$x = 4$$

**9. [10 points] Solve.**

$$\frac{1}{5} + \frac{3}{x+3} = \frac{9}{2x+6}$$

$$(10x + 30) \frac{1}{5} + (10x + 30) \frac{3}{x+3} = (10x + 30) \frac{9}{2x+6}$$

$$(2x + 6) + 10(3) = 5(9)$$

$$2x + 36 = 45$$

$$x = 4.5$$